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ORBIT DETERMINATION FROM MINITRACK OBSERVATIONS

by

R. H. Gooding

MINISTRY OF AVIATION
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SUMMARY

Although Minitrack observations are only accurate to about 1 minute of arc, accurate orbits have been obtained for a number of satellites. This is due to the excellent global coverage of the NASA Minitrack network. The accuracy obtained for eccentricity is typically about 10^{-5}, and comparable values are obtained for the other orbital elements.

The main source of observational error is thought to be inadequate correction for ionospheric refraction. Apparent error arises through deficiencies in the orbital model, namely, inadequate representation of satellite perturbations due to the Earth's tesseral harmonics and to atmospheric drag.

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Departmental Reference: Space 178

CONTENTS

1 INTRODUCTION

Radio tracking of satellites has an advantage over radar tracking, in that power requirements are much less, and an advantage over optical observation, in that conditions of light and darkness are irrelevant. The main disadvantage, in comparison with the other two sources of observations, is that there must be a transmitter on board the satellite.

Two possible techniques exist for extracting observational data from radio tracking. They may be employed simultaneously but this is not normal. The first technique - measuring range rate by the Doppler principle - has been described by Newton[1]. The alternative is to measure direction cosines by the interferometry principle; this is the Minitrack technique.

Though the French have set up two Minitrack stations, and ESRC are setting up another, the only data so far seen at R.A.E. are from the STADAN network of NASA. This, presently, consists of a dozen stations as shown in Fig.1.

2 THE MINITRACK PRINCIPLE

If two aerials are set at the ends of a baseline, an observed satellite will, in general, be nearer to one than to the other. The two received signals interfere and the difference in phase gives a direct measurement of the direction cosine of the satellite relative to the baseline. Four aerials, two on a north-south baseline and two on an east-west baseline, will give two direction cosines - that is, a complete specification of direction.

In fact, however, a typical Minitrack station[2,3] has 13 aerials. The four aerial system is duplicated: one system, the 'equatorial system', is much more electrically sensitive in the north-south direction than in the east-west direction; for the other system, the 'polar system', the reverse is true. In addition, the main baselines, being 55 wavelengths (about 120 m) long, lead to phase ambiguities. These ambiguities are resolved by supplementing the two 'fine' systems by 'medium' and 'coarse' systems, requiring five further aerials. Since these aerials are isotropic they serve both the polar and the equatorial systems.

Before a satellite pass, the equatorial or polar system is selected, according to the expected ground-track. This is equivalent to erecting a detection fence through which the satellite, on average, takes about 30 seconds to pass. The NS and EW, fine, medium and coarse interferometers take 150 measurements of phase difference during this time; these are recorded on paper tape and transmitted to NASA (Goddard Space Flight Center).

The raw data from all the stations are processed by NASA and reduced to two observations, about 15 seconds apart, for each station/pass. Corrections for ionospheric refraction are incorporated at this stage. The resulting data, from all satellites currently tracked, are distributed to interested parties, including R.A.E., every few weeks.

3 ORBIT DETERMINATION

Naturally, NASA analyse their own data for all satellites tracked. Even when they are not themselves interested in a particular orbit they must do this to provide tracking predictions. Though orbital elements may be obtained from NASA, they are not formally published. Since 1963 they have been derived by using the orbital theory of Brouwer[16]. This means, effectively, that they are derived from a double averaging of osculating elements: first with respect to mean anomaly to remove short-periodic (J_2) perturbations, and second with respect to argument of perigee, ω, to remove long-periodic perturbations; long-periodic terms are removed on the assumption that they are composed of terms in $\sin \omega$, $\cos \omega$, $\sin 2\omega$ etc.; terms in J_2^2, J_3, J_4 and J_5 are accounted for, with, until recently*, the following values taken: $J_2 = 1082.19 \times 10^{-6}$, $J_3 = -2.285 \times 10^{-6}$, $J_4 = -2.123 \times 10^{-6}$, $J_5 = -0.232 \times 10^{-6}$.

Orbits for four satellites have been determined at R.A.E. The theory of Merson[4], in which short-periodic perturbations are catered for by use of smoothed elements, is the basis of a differential correction programme written for the Pegasus computer[5]. Long-periodic perturbations are treated as if secular; over a period of 3 or 4 days this does not involve any noticeable loss in accuracy.

4 ORBITS OF TIROS 7 AND ALOUETTE 1

The orbit of Tiros 7 (1963-24A) has been determined, during a ten-day period, by Diana Scott[6], and the orbit of Alouette 1 (1962 $\beta\alpha 1$) during an eight-day period, by Hiller[7]. The orbital inclinations are, respectively, 58° and 80°. The small-eccentricity theory of Cook[8] has been used in interpreting the orbits, and orbital elements have been compared with values provided by NASA. The eccentricity, e, was less than 0.005 for both satellites and it was decided to analyse the Tiros orbit just after a maximum in an eccentricity cycle - eccentricity being maximum when ω, the argument of perigee, was 90° - and the Alouette orbit around a minimum in the cycle, i.e. near $\omega = 270^\circ$. Differences between R.A.E. and NASA elements were expected to be largest at eccentricity maxima and minima.

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*Currently the values used are 1082.48×10^{-6}, -2.56×10^{-6}, -1.84×10^{-6} and -0.06×10^{-6}.

Results showed that the small-eccentricity theory, in which elements e and ω are replaced by e cos ω and e sin ω, describes this type of orbit much better than the ordinary sinusoidal theory of long-periodic motion. This is illustrated in Fig.2 which shows how well the theoretical curve, drawn over a complete cycle, fits the observed values of eccentricity for Alouette 1. A sinusoidal curve gives only an approximate fit. NASA values of eccentricity, shown over a complete cycle of perigee, were presumably obtained by subtracting a sinusoidal term from the normal eccentricity, the amplitude of this term (about 0·8 times the amplitude of the dashed curve in Fig.2) being given by the NASA values of J_3 and J_5 already quoted - the contribution from J_5 is actually negligible. Correction of the NASA eccentricities on this basis, however, gives values which still appear to be inconsistent with the theoretical behaviour.

Average accuracies (s.d.'s) for computed orbital parameters are listed in the Table below. Figures for Ariel 2, discussed in the next section, are included. The parameters n_1 and n_2 are linear and quadratic coefficients in the mean-motion polynomial.

Since both Tiros 7 and Alouette 1 were in virtually drag-free orbits, Tiros 7 being at a height of 635 km and Alouette 1 at a height of 1000 km, the very low s.d. for semi-major axis really does provide a valid estimate of random error for these two satellites, though it does not for Ariel 2 which is much more affected by drag[9]. There may, however, be a systematic error as large as 20 m, arising from error in the assumed value (398602 km^3 sec^{-2}) of the earth-mass × gravitation constant.

<u>Accuracies of computed orbital elements</u>

	Tiros 7	Alouette 1	Ariel 2
Semi-major axis, a	0·4 m	1 m	1 m
Eccentricity, e	6×10^{-6}	14×10^{-6}	8×10^{-6}
Inclination, i	0°·0006	0°·0012	0·0005
R.A. of node, Ω	0°·0008	0°·0013	0·001
*Argument of perigee, ω	0°·0005	0°·0012	0°·0005
Time at node, t_0	20 msec	25 msec	30 msec
n_1 in deg/(100 days)^2		17	12
n_2 in deg/(100 days)^3			1500

*For meaningful comparison with other elements, the s.d.'s of ω have been multiplied by e.

5 ORBITS OF ARIEL 1 AND ARIEL 2

The orbit of Ariel 1 (1962 o1) was the first to be determined at R.A.E. from Minitrack data (when the computer programme was still under development). The object in determining this orbit was to assess the accuracy that was likely to be attainable for the satellite U.K.2 which had not then been launched. Whereas NASA provided the experimenters with definitive elements for Ariel 1, it was intended that R.A.E. should do this for U.K.2.

U.K.2 was launched on 27th March 1964 and was then rechristened Ariel 2 (1964-15A). The orbital inclination is 51°·6; initial perigee and apogee heights were 285 km and 1360 km. The orbit was determined every $1\frac{3}{4}$ days (at 25 node intervals, to be exact) for a period of 12 months; satellite position can be computed, using the elements listed in Ref.9, for any instant during this period, with the error never exceeding a km or so. Average e.d.'s have been listed in the Table above; inclination is determined so accurately (to about 0°·0005 in s.d.) that the change in i over a year has been used by King-Hele and Miss Scott[10] in a study of the rotation of the upper atmosphere, even though this change was much smaller (about 0°·005) than the corresponding change for satellites in lower orbits. The amplitude of the oscillation in the sinusoidal component of the eccentricity variation has been used by King-Hele et al[11], in conjunction with data from other satellites, to obtain values for the earth's odd zonal harmonics, J_3, J_5, etc. The rate of change of Ω has been used, similarly, by Smith[12] in an evaluation of the even harmonics.

A plot of eccentricity over 12 months is given in Fig.3. The weekly values of NASA are also plotted; the long-periodic oscillation, of amplitude 0·001, has clearly been removed. On restoring this oscillation, the corrected values of NASA lie on the R.A.E. curve. It is useful to remove the secular trend and part of the oscillation from the eccentricity, in order to exhibit its behaviour on a ten-times-larger scale. Fig.4 shows the result; values are represented by lines of length 2 e.d.'s, the average value of an e.d. being 0·00001.

6 AN UNEXPLAINED DISCREPANCY

To give the Ariel 2 experimenters their requested accuracy - satellite position to $\frac{1}{2}$ km whenever possible - it was found necessary to remove an apparent along-track error in satellite computed position. This error was sinusoidal, with amplitude 650 m and period just under half a day. It was diagnosed as due to the effect of certain of the tesseral harmonics of the Earth's gravitational field; due in fact to those coefficients $J_{n,m}$ for which m = 2 and n ($\geqslant$ m) is even.

Now approximate values of $J_{2,2}$, $J_{4,2}$, $J_{6,2}$ and $J_{8,2}$ are available: They have been computed by fitting to observed perturbations for a number of satellites, though not for any with orbits determined from Minitrack observations. These values predict an along-track oscillation for Ariel 2, of the period observed but of amplitude only half that observed. Expressing the along-track oscillation as a time error in fact (where $7\frac{1}{2}$ km is equivalent to 1 s), the amplitudes contributed by Guier and Newton's values[13] of $J_{2,2}$, $J_{4,2}$, $J_{6,2}$ and $J_{8,2}$ should be respectively $0^s{\cdot}051$, $0^s{\cdot}017$, $0^s{\cdot}004$ and $0^s{\cdot}002$, giving (allowing for the different phases) a total of $0^s{\cdot}051$. The observed amplitude, however, is $0^s{\cdot}088$, as shown in Fig.5.

Thus there is a discrepancy of which the explanation is not known. The Guier-Newton value of $J_{2,2}$ has been confirmed, to within a few per cent, by Allan and Piggott[14] using results from the synchronous satellites Syncom 2 and Syncom 3, and it would be inconceivable that this value should be wrong by a factor of nearly 2. It is interesting to note, however, that if we <u>do</u> make this assumption, the observed along-track oscillation is obtained not only for Ariel 2 but also for the satellite Ogo 2 (1965-81A) on which some preliminary analysis, using Minitrack observations, has been done. The orbital inclination of this satellite is very different ($87^\circ{\cdot}4$) so that the effects of $J_{2,2}$, $J_{4,2}$ etc. should combine differently for the two satellites.

Several possible explanations have been suggested, but each has had to be rejected. The most likely is still that the true values of $J_{4,2}$, $J_{6,2}$, etc., are much bigger than those in Ref.13. The author is not aware of any other perturbation which would be of the right period, viz $\pi/(\dot{\theta} - \dot{\Omega})$, where θ is the sidereal time and Ω is the right ascension of the satellite node; for Ariel 2 it is known very accurately that the observed perturbation has this period, since the orbit has been analysed for a full year.

7 ACCURACY OF MINITRACK OBSERVATIONS

Although the quantities observed by the Minitrack stations are direction cosines, these are converted (in R.A.E. analysis) into azimuth and elevation before use. The differential correction programme works on the assumption that the errors in all observations are random, uncorrelated and normally distributed: furthermore, that $\cos E \times \sigma(A) = \sigma(E) = 1'$, where A and E denote azimuth and elevation; time errors are ignored. The standard deviation assigned (1 minute of arc) is a round figure based on experience. At the end of every complete orbit determination an a-posteriori estimate of accuracy is computed and included, as a factor, in the estimated standard deviations of the orbital elements. This a-posteriori estimate is normally between 1' and 2'.

The estimated instrumental accuracy of the Minitrack system, when it was designed, was about 20", the unit of resolution being about 5" (corresponding to 0.36 electrical degree in raw phase measurement). After smoothing, the random error should be negligible so that it is of interest to know why the a-posteriori accuracy of an observation is as bad as it is - it is interesting to note, in this context, that the best visual observers can equal this performance of 1 or 2 minutes of arc.

Error arises from the following sources: phase drift, ionospheric refraction, station survey, timing; apparent error (as shown by residuals) will also arise from deficiencies in the orbital model used in the analysis.

Phase drift may build up to 30" over a six-month period, after which there is a recalibration. Errors from ionospheric refraction are potentially much larger than this. However, large errors are avoided by NASA who (a) correct for ionospheric refraction using I.T.S.A. (Boulder) predictions and (b) normally restrict disseminated data to observations when the elevation is greater than 70°.

The stations have recently been surveyed relative to the Fischer ellipsoid and errors in their position should not exceed 100 m. Timing - local timing, at any rate - is thought to be good to a millisecond or so.

Thus, of the genuine observational errors, only those due to ionospheric refraction are likely to be large enough to account for the residuals obtained. The author would have thought that even these, in general, would not be large enough.

Two deficiencies in the model must be mentioned. The first concerns the tesseral harmonics of the Earth's gravitational field. As has been explained, allowance is made for an along-track perturbation of half-day period. Apart from this, however, the tesseral harmonics have so far been neglected. Typically this may lead to residuals of 1', or more for close satellites, but no pattern in the residuals has been detected, of the type which would indicate that this is the dominant cause of (apparent) observational error.

The other deficiency concerns the representation of drag perturbations. This is inevitable, unlike the situation for tesseral harmonic effects which will eventually be programmed, since the atmosphere fluctuates from hour to hour. For a satellite like Ariel 2, though a cubic polynomial is fitted to the mean anomaly to represent the perturbation as far as possible, the residual effects could be large enough to account for the observed residuals. For satellites with much higher perigees, like Tiros 7 and Alouette 1, the residual drag effects should be small; the a-posteriori estimate of accuracy for these satellites is still[6,7] about 1'.3, however.

8 ACCURACY USING A MINIMAL NETWORK

Since ESRO, or the U.K. if a national programme were contemplated, will not normally have access to the STADAN network, it is important to know how the accuracy of orbit determination is affected when only two or three stations are providing observations. Some accuracy assessment studies along these lines have been carried out at R.A.E.

Consider, as an example, the case of a polar orbit with perigee height 300 km and apogee height 1000 km, with the satellite observed by two stations, one in S.E. England or Belgium and the other in Southern Australia. Merson has shown[15] that at each station there should be an average of about 1 pass per day for which the satellite reaches an elevation greater than 50°. Over three days this would give 6 passes, i.e. 12 observations, assuming 2 per pass. For a particular choice of longitude crossing at the central node, an accuracy assessment was made; with this choice there was only 1 pass from the first station, but 5 for the second. Assuming an accuracy of $1\frac{1}{2}'$ for the observations and a seven-parameter model, accuracies (s.d.'s) of orbital parameters worked out as follows: $\sigma(a) = 4$m, $\sigma(e) = 10^{-5}$, $\sigma(i) = \sigma(\Omega) = 0°{\cdot}0015$, $e\,\sigma(\omega) = 0°{\cdot}001$, $\sigma(t_0) = 80$ msec and $\sigma(n_1) = 40$ deg (100 days)$^{-2}$. With these accuracies a satellite ephemeris to an accuracy of 1 km should normally be attainable.

As a practical example of what can be achieved, the orbit of Ariel 2, over the first $3\frac{1}{2}$ days after launch, has been re-analysed, using observations from Winkfield (England) and Johannesburg only. With 12 observations - all at elevation greater than 50° - and a seven parameter model (no n_2) the following s.d.'s were obtained: $\sigma(a) = \frac{1}{2}$ m (largely fictitious due to drag fluctuations), $\sigma(e) = 10^{-5}$, $\sigma(i) = 0°{\cdot}0007$, $\sigma(\Omega) = 0°{\cdot}0017$, $e \times \sigma(\omega) = 0°{\cdot}0007$, $\sigma(t_0) \approx 54$ msec and $\sigma(n_1) = 12$ deg (100 days)$^{-2}$. Estimated ephemeris errors were less than $\frac{1}{2}$ km, though it is thought that errors would be larger than this due to biases in the observations.

As an exercise in what is probably the ultimate in reducing the network, the example just quoted was repeated, with 7 observations, from Johannesburg only, using a six-parameter model (neither n_1 nor n_2). The following were the s.d.'s: $\sigma(a) = 1\frac{1}{2}$m, $\sigma(e) = 10^{-4}$, $\sigma(i) = 0°{\cdot}005$, $\sigma(\Omega) = 0°{\cdot}003$, $e \times \sigma(\omega) = 0°{\cdot}005$ and $\sigma(t_0) = 0{\cdot}14$ sec. Estimated ephemeris errors rose to 4 km.

This last example is somewhat exceptional, as will now be seen. There are two complementary requirements in order that orbital analysis shall yield good orbital parameters. The first is good coverage in <u>time</u>. Here there is usually no difficulty. To obtain a good value of mean motion (equivalent to

semi-major axis) it is desirable to have two passes $3\frac{1}{2}$ days apart, say, with the same part of the orbit observed during each pass. (A third pass, half way between, will allow a value of n_1 to be computed, etc.) For the other elements, however, it is essential to have a good coverage in angl (in true anomaly, say) and this is the second requirement. This is rarely available from one station, but in the example quoted Johannesburg observed one south-going pass as well as a number of north-going passes. The 7 observations used consisted, in fact, of 2 from a north-going pass, 3 from a south-going pass $1\frac{1}{2}$ days later, and then 2 from a north-going pass $1\frac{1}{2}$ days after that.

9 CONCLUSIONS

Orbits of a number of satellites have been analysed at R.A.E., using Minitrack data kindly supplied by NASA. Results have been very satisfactory, with average accuracies (s.d.) including 10^{-5} in eccentricity and $0°{\cdot}001$ in inclination and right ascension of the node. These accuracies are due to the excellent global coverage of the STADAN Minitrack network and to the fact that radio observations are not hampered by visibility restrictions.

The accuracy of the observations themselves is disappointing. At 1 or 2 min of arc it is no better than can be achieved by the best visual observers. Errors, real or apparent, may be attributed to a number of sources: inadequate correction for ionospheric refraction, irregularities in atmospheric drag, incomplete representation of tesseral harmonics, phase drift, station survey and timing. The main sources are likely to be the first three of these, but it should be possible to reduce the effects, even of these, below the 1' level.

To obtain well-fitting orbits for the satellite Ariel 2, it has been found necessary to use an unnaturally high value of the tesseral harmonic $J_{2,2}$. The reason for this is not yet known.

ACKNOWLEDGMENT

I am grateful to Dr. J.W. Siry of NASA for some helpful comment.

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Fig.1

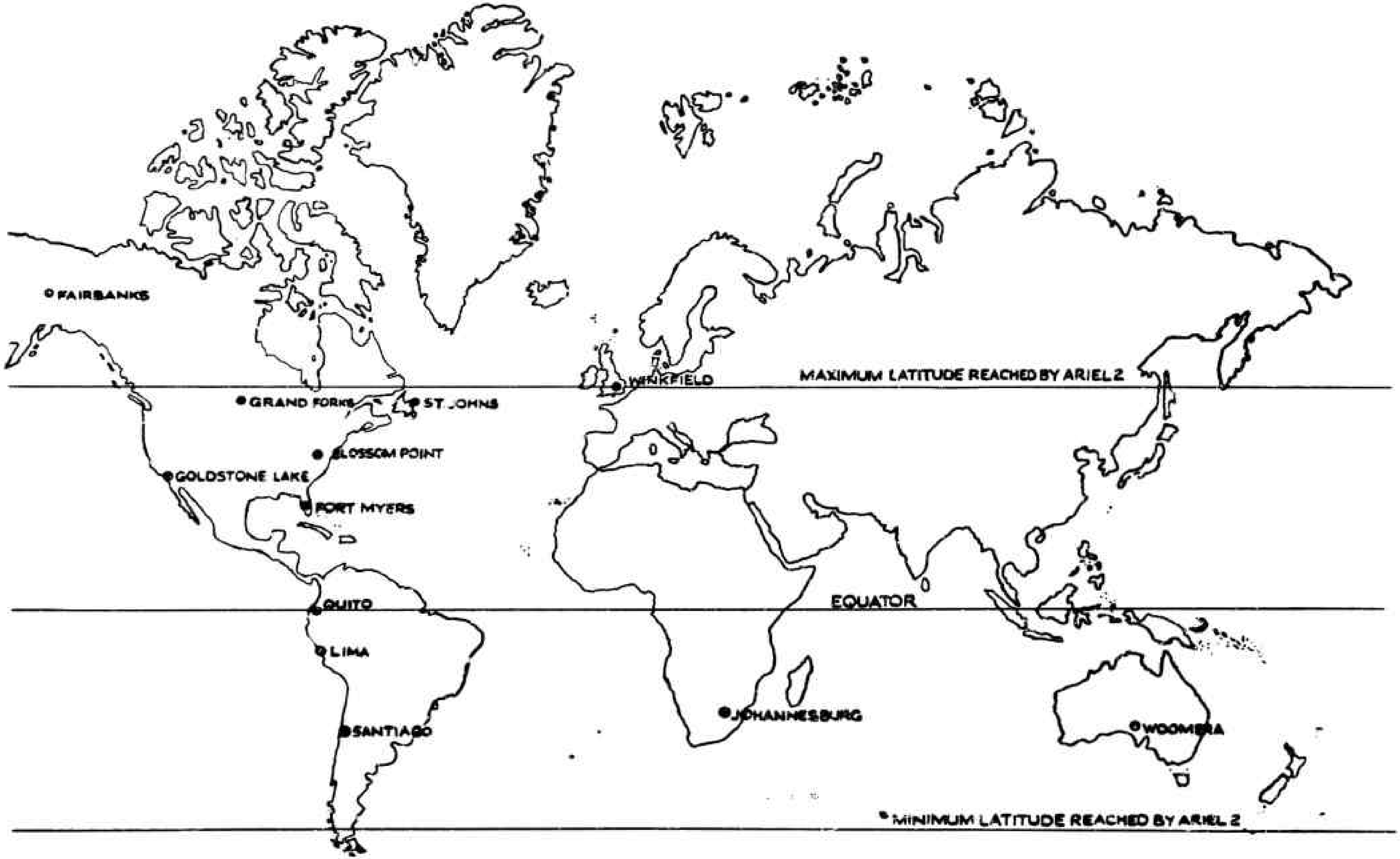

FIG.1 THE STADAN NETWORK OF MINITRACK STATIONS

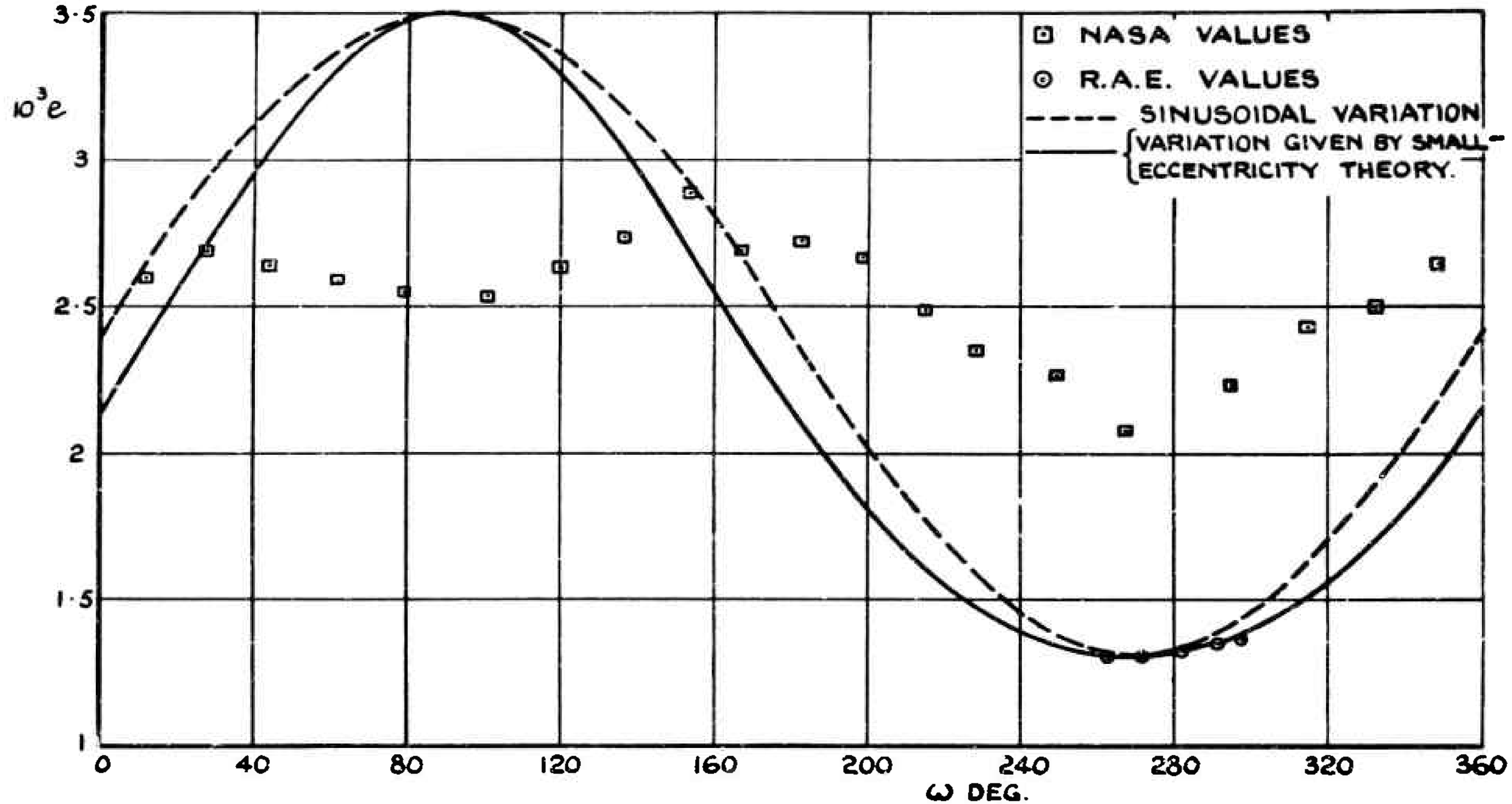

FIG. 2 ECCENTRICITY OF ALOUETTE I vs ARGUMENT OF PERIGEE

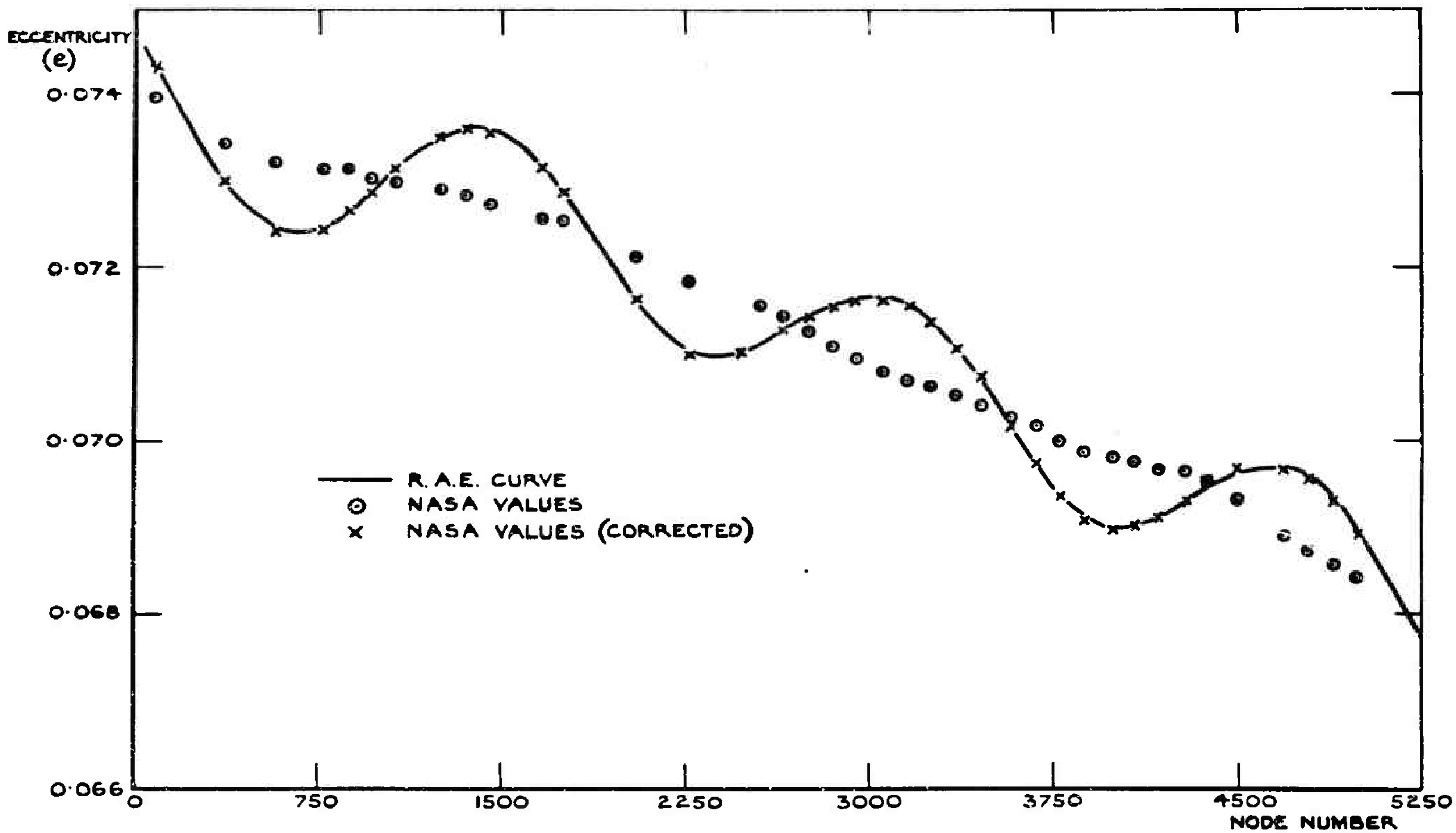

FIG. 3 ECCENTRICITY OF ARIEL 2 — NASA AND R.A.E. VALUES COMPARED

Fig.4

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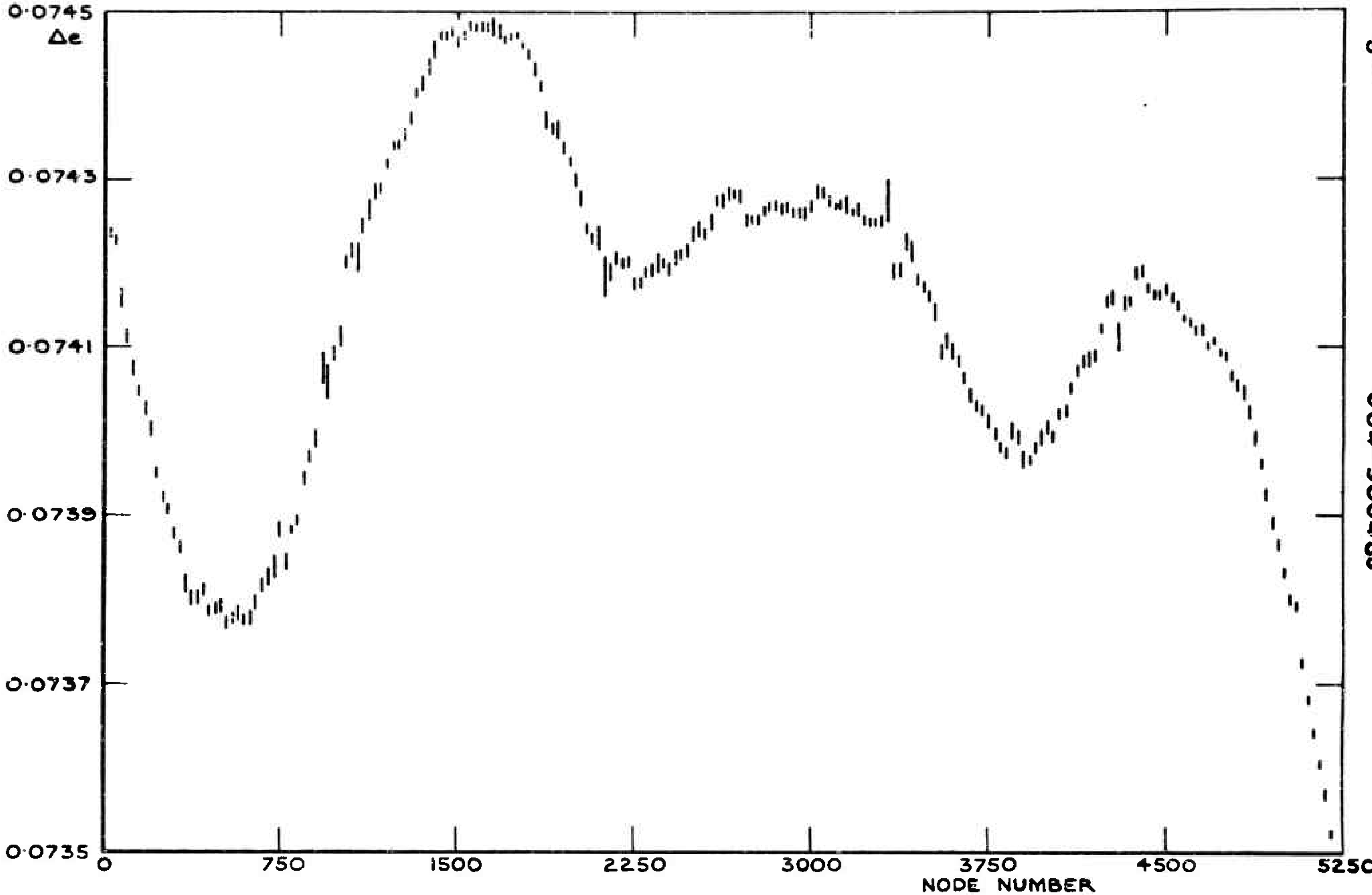

FIG. 4 ECCENTRICITY OF ARIEL 2 – LINEAR AND SINE TERMS REMOVED

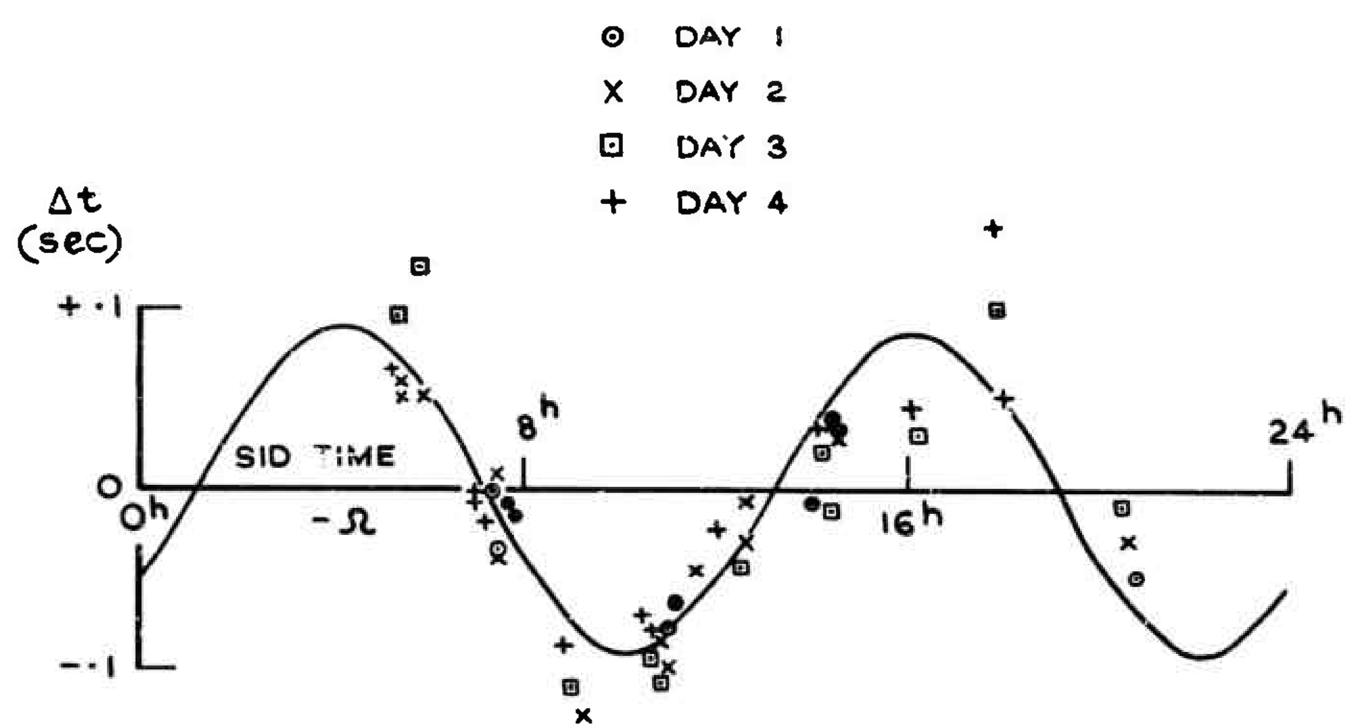

FIG. 5 APPARENT TIME ERROR (ARIEL 2)